Modeling the transmission of beta rays through thin foils in planar geometry

D. Stanga, P. De Felice, J. Keightley, M. Capogni, E. Ionescu

A mathematical model of electron transport in planar geometry is developed. The model is based on the plane source concept. The efficiency of plane sources is computed using Monte Carlo method. A simple function for the plane source efficiency is obtained by curve fitting. Applications of the mathematical model are also presented.

1. Introduction

The transmission of beta rays through thin absorbers in planar geometry is quite different from the case of collimated beams. This is due to the fact that the emission of beta particles is isotropic and the majority of trajectories through the absorber are longer than the thickness of the absorber. The geometry effect on experimentally determined transmission curves (Jansen and Klein, 1996) and a heuristic approach for calculating roughly the attenuation of beta-rays in thin absorbers in planar geometry have already been reported (Haemers et al., 2007). Monte Carlo simulation of the electron transport through thin foils in planar geometry was also performed for determining the efficiency of large-area beta sources (Berger, 1998; Svec et al., 2006; Stanga et al., 2011).

The mathematical modeling of the transmission of beta rays through thin foils in planar geometry is useful for a range of applications such as standardization of large-area sources (ISO, 2010; Berger et al., 1996), calibration of contamination monitors (IEC, 2002), evaluation of surface contamination (ISO, 1988; ISO, 1996) and gross beta counting (ISO, 1992; Pujol and Suarez-Navarro, 2004). By mathematical modeling, the problems from these areas are translated into tractable mathematical formulations whose theoretical and numerical analysis provides insight, answers and guidance useful for these applications.

This paper deals mainly with the modeling of the transmission of beta rays through thin foils in planar geometry based on the plane source concept, using Monte Carlo simulation of electron transport and least squares fitting. Applications of modeling results for calculating the efficiency of large-area beta sources, transmission coefficient of beta rays through thin foils and the beta detection efficiency of large-area detectors used in surface contamination measurements are also presented.

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transmission coefficient of beta rays through thin foils and the beta detection efficiency of large-area counters. These are powerful tools with many applications in the areas mentioned above. Thus, the integral equation for calculating the source efficiency provides a simple relationship between the surface emission rate and the activity of large-area beta emitting sources. This is in contrast with the general opinion that there is not a simple and known relationship between these quantities (ISO, 2010). The same integral equation can be used for evaluating the surface contamination by calculating the efficiency of beta contamination sources. The standard ISO 7503-1 provides two suggested default values for the efficiency of beta contamination sources but they do not have a strong theoretical basis (ISO, 1988).

Novel methods of measurement can be developed by using the integral equations for calculating the source efficiency and the transmission coefficient. A new method, based on these equations, for determining the activity of large-area beta reference sources constructed from anodized aluminum foils has already been reported (Stanga, 2014). The integral equation for calculating the detection efficiency can be used for investigating under which conditions the calibration of surface contamination meters on the basis of activity per unit area is useful.

In September 2014 the three year duration Joint Research Project "MetroDecom-Metrology for decommissioning nuclear facilities" started in the frame of the European Metrology Research Programme (EMRP). One of specific objectives of the project is to improve the accuracy and traceability of surface beta contamination measurements. The modeling of the transmission of beta rays through thin foils in planar geometry is the starting point in the achievement of this objective.

2. Transmission of beta-rays emitted by plane sources through thin foils

2.1. Efficiency of plane sources

We consider the large-area source shown in Fig. 1 without the covering foil. In this planar geometry the source substratum of thickness $\Delta$ is placed on a backing plate both being constructed from the same material of circular shape having identical radii. The activity is incorporated into the top surface of the source substratum resulting in a source which has a thin active layer and a circular shape. The backing plate is thick and the radius of the active layer is small to prevent emission of the beta radiation through the back and the side of the source. We used a circular type source but the results of this paper remain valid for any type of planar source.

For a point source of infinitesimal volume assumed to be located at the position $(x, y, z)$, its emission rate in $2\pi$, $E(x, y, z) \, dx \, dy \, dz$, is defined as the rate of beta particles that emerge from the top surface of the large-area source in a $2\pi$ solid angle. The emission in $2\pi$ of the source results from the interplay of two factors. On the one hand, emission is increased due to the backscattering of beta particles by the backing plate, while on the other hand, the emission is decreased due to the absorption of beta particles by the substratum material. The efficiency of the point source is defined as the ratio between its emission rate $E(x, y, z) \, dx \, dy \, dz$ and its activity $\lambda(x, y, z) \, dx \, dy \, dz$ (assuming that the emission probability of beta particles is 100%). It is evident that the efficiency of the point source does not depend on the coordinates $y$ and $z$. Under these conditions, the efficiency, $\varepsilon_p(x)$, of the plane source located at the depth $x$ can be defined as

$$\varepsilon_p(x) = \frac{E_p(x)}{A_p(x)} \quad (1)$$

where $A_p(x) = \iint A(x, y, z) \, dy \, dz$, $E_p(x) = \iint E(x, y, z) \, dy \, dz$ ($S$ is the surface of the plane source), $A_p(x) \, dx$ and $E_p(x) \, dx$ represent the activity and the emission rate in $2\pi$ of the plane source. In case that the substratum material is different from the material of the backing plate, $\varepsilon_p(x, \Delta)$ depends on both $x$ and the thickness $\Delta$ of the substratum material. This is due to the fact that the electrons are backscattered by a double-layered material composed of a backing plate (with the thickness higher than the backscattering saturation thickness) and a thin layer of variable thickness containing the substratum material (see chapter 3). In this case, Eq. (1) becomes

$$\varepsilon_p(x, \Delta) = \frac{E_p(x, \Delta)}{A_p(x)} \quad (2)$$

In practice, large-area beta sources are often constructed as it is shown in Fig. 2. This planar geometry is equivalent with the geometry from Fig. 1 when the substratum and backing plate are made from the same material. In case that these materials are different, the efficiency of a point source located at the position $(x, y, z)$ depends on the coordinates $y$ and $z$ located near the source edge. Consequently, the efficiency of the plane source from the depth $x$ depends on the coordinates $y$ and $z$. However, this edge effect can be neglected if the atomic numbers of materials are close and/or the source radius is much longer than the maximum range of beta-rays in the substratum material.

As mentioned above, Eq. (1) is valid for nuclides that emit beta radiations with emission probability of 100% such as $^{14}$C, $^{147}$Pm, $^{60}$Co, $^{36}$Cl and $^{90}$Sr-$^{90}$Y. In case of nuclides that emit both beta particles having a continuous energy spectrum and conversion electrons having discrete energies $E_i$ with emission probabilities $f_i$ $(i = 1, 2, ..., n)$, the plane source efficiency can be written as

$$\varepsilon_p(x) = \frac{E_p(x)}{f_iA_p(x)} = \frac{E_b(x) + E_c(x)}{f_iA_p(x)} = \frac{f_iE_{bp}(x) + f_iE_{pec}(x)}{f_i} \quad (3)$$

where $E_b(x)$, $f_i$ and $E_{bp}(x)$ are, respectively, the emission rate in $2\pi$, the emission probability and the plane source efficiency corresponding to beta particles, $E_{ce}(x)$, $f_i$ and $E_{pec}(x)$ are, respectively, the emission rate in $2\pi$, the total emission probability and the plane source efficiency corresponding to conversion electrons, $f_i = f_{b,c} + f_{c}$ and $E_p(x) = E_b(x) + E_c(x)$. It should be noted that Eq. (3) reduces to Eq. (1) when $f_b$ = 1 and $f_c$ = 0. The efficiency, $E_{ce}(X)$, is given by

$$E_{ce}(X) = \frac{E_{ce}(x)}{f_{ce}A_p(x)} = \frac{1}{f_{ce}} \sum_{i=1}^{n} f_iE_{bp}(X) \quad (4)$$
where \( f_p = \sum f_i \) and \( \epsilon_p(X) \) is the plane source efficiency corresponding to conversion electrons of energy \( E_x \). The radionuclide \( ^{137}\text{Cs} \) is an example of a nuclide that emits both beta radiations and conversion electrons (via \( ^{137}\text{Ba} \) metastable state with \( T_{1/2} = 2.55 \text{ min} \)). These electrons have three discrete energies \( E_1 = 624 \text{ keV}, E_2 = 656 \text{ keV} \) and \( E_3 = 660 \text{ keV} \) with emission probabilities \( f_1 = 0.0762, f_2 = 0.0142 \) and \( f_3 = 0.0033 \) (Bé et al., 2006; Unoki and Stanga, 2013).

Taking into account the covering foil of thickness \( s \) shown in Fig. 1, the efficiency of the plane source located at the depth \( x \), \( \epsilon_p(X, \Delta, s) \), can be defined as

\[
\epsilon_p(X, \Delta, s) = \frac{E_p(x, \Delta, s)}{E_p(x, \Delta)} = \frac{\epsilon_p(X, \Delta, s)}{\epsilon_p(X, \Delta)} \quad \text{(5)}
\]

where the emission rate in \( 2\pi \) of the plane source (the rate of beta particles that emerge from the top surface of the covering foil in a \( 2\pi \) solid angle), \( E_p(X, \Delta, s) \), depends on \( x, \Delta \) and \( s \). The transmission coefficient of beta rays emitted by the plane source through the foil of thickness \( s \) is defined as

\[
\epsilon_p(X, \Delta, s) = \frac{E_p(x, \Delta, s)}{E_p(x, \Delta)} \quad \text{(6)}
\]

and represents the fraction of beta particles transmitted through this foil.

The concept of plane source, introduced by Berger et al. (1996), is useful in treating the efficiency of large-area beta sources and the transmission of beta rays through thin foils in planar geometry because the plane source efficiency can easily be calculated by Monte Carlo method and depends only on the source depth \( x \) and the thickness, \( \Delta \), of the source substrate.

### 2.2. Computing the efficiency of plane sources by Monte Carlo method

Electron transport calculations by the Monte Carlo method were carried out using the Pencil code from the simulation package PENELOPE (Baro et al., 1995). This code simulates electron–photon showers in multilayered cylindrical structures. The reliability of the simulation package PENELOPE for simulating the electron transport was confirmed in different benchmark comparisons and comparative studies (Sempau et al., 2003; Vilches et al., 2007). In the Pencil code, the simulation of electron tracks is performed by means of a mixed (class II) algorithm. A detailed description of cross sections and simulation methods adopted in

<table>
<thead>
<tr>
<th>Backing plate: aluminum</th>
<th>Source substrate: aluminum</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^{14}\text{C} )</td>
<td>( ^{14}\text{C} )</td>
</tr>
<tr>
<td>( x ) (mg/cm²)</td>
<td>( y_p(x,k) )</td>
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<tr>
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<td>0.7205</td>
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<tr>
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<td>0.6345</td>
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<tr>
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<tr>
<td>0.675</td>
<td>0.3270</td>
</tr>
<tr>
<td>1.080</td>
<td>0.2843</td>
</tr>
</tbody>
</table>

| 1.620 | 0.2222 | 1.620 | 0.2797 | 5.600 | 0.1050 | 52.500 | 0.1003 | 52.500 | 0.1003 |

Table 1: Values of \( \epsilon_p(X,k) \) computed by Monte Carlo method for five nuclides and different values of \( x \) of the source depth using aluminum and Mylar as materials for both the backing plate and the source substrate.
where $\Delta x$ for $137$Cs plane sources computed by Monte Carlo method for different values $x$ of the source depth using aluminum and Mylar as materials for both the backing plate and the source substrate.

These parameters were computed using Monte Carlo data from Table 2. Table 3 shows the values of $\epsilon_3(x, \Delta)$ for different values of $x$. Aluminum and Mylar were used for both the backing plate and the substrate. All Monte Carlo data were computed with relative standard uncertainties smaller than 0.3%.

Electron transport calculations were also carried out for determining the efficiency of $^{36}$Cl plane sources using aluminum as material for the backing plate and Mylar as source substrate. In Table 3 are shown the values of $\epsilon_3(x, \Delta)$ for different values of $x$ computed for $\Delta = 15.6$ mg/cm$^2$.

### Table 2

<table>
<thead>
<tr>
<th>$x$ (mg/cm$^2$)</th>
<th>$\epsilon_3(x, \Delta)$</th>
<th>$x$ (mg/cm$^2$)</th>
<th>$\epsilon_3(x, \Delta)$</th>
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<tr>
<td>0.00</td>
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<td>0.56</td>
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<td>9.80</td>
<td>0.3816</td>
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<tr>
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<td>0.5834</td>
<td>12.50</td>
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<tr>
<td>1.40</td>
<td>0.5686</td>
<td>15.60</td>
<td>0.3246</td>
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</tbody>
</table>

### 2.3. Least square fitting of Monte Carlo data

The Monte Carlo data were fitted with deviations of less than 0.5% by the following polynomial function in the variable square root of $x$.

$$\begin{align*}
\epsilon_3(x) = \left( a_0 + a_1 x^{0.5} + a_2 x^{2.0} + a_3 x^{3.0}\right)
\end{align*}$$

where $a_0$, $a_1$, $a_2$, $a_3$, and $a_i$ are fitting parameters. In Table 4 are shown the values of fitting parameters for $^{14}$C, $^{147}$Pm, $^{60}$Co, $^{36}$Cl, and $^{90}$Sr-$^{90}$Y plane sources which were computed using Monte Carlo data from Table 1. One can see from Table 4 that $\epsilon_3(x)$ depends not only on $x$ but also on materials used for the backing plate and the source substrate. It is also evident that $a_0 = (1 + \eta_s)/2$ where $\eta_s$ is the saturation backscattering coefficient of the plate material (see chapter 3).

In the case of $^{137}$Cs plane sources, the fitting parameters, $a_i (i = 0, 1, 2, 3, 4, 5)$, were calculated by means of Eq. (3) as follows

$$\begin{align*}
a_i = f_i^t \Delta a_i
\end{align*}$$

where $a_0$, $a_1$, $a_2$, $a_3$ are, respectively, the fitting parameters corresponding to $\epsilon_3(x)$ for $^{14}$C, $^{147}$Pm, $^{60}$Co, $^{137}$Cs, $^{36}$Cl and $^{90}$Sr-$^{90}$Y plane sources and $x \in [0, 7]$ mg/cm$^2$ using aluminum as material for both backing plate and source substrate.

Monte Carlo data from Table 3 were also fitted with the function expressed by Eq. (7). The following values of the fitting parameters were obtained: $a_0 = 0.67866$, $a_1 = -0.10582$, $a_2 = 0.00017$, $a_3 = 0.00261$, $a_4 = -0.00083$ and $a_5 = 0.00011$. As one can see, $a_0 = (1 + \eta_s^{(am)}(\Delta))/2$ where $\eta_s^{(am)}(\Delta)$ is the backscattering coefficient at saturation of the double-layered material composed of the aluminum plate and the Mylar substrate of thickness $\Delta = 15.6$ mg/cm$^2$ (see Eq. (11)).

### 3. Backscattering of beta-rays in planar geometry

The backscattering coefficient is defined as the ratio of the number of backscattered beta particles to the number of beta particles.
particles hitting the surface of the backing plate. The backscattering coefficient increases with increasing plate thickness until saturation is reached. The largest backing layer, at which the backscattering coefficient reaches its maximal value, is called the thickness of saturation. The backscattering coefficient corresponding to values of the backing plate thickness higher than the saturation thickness is called saturation backscattering coefficient.

The backscattering coefficient in planar geometry was computed by means of the Pencyl code. Table 6 shows the values of the backscattering coefficient, $\eta(x)$, for different values, $x$, of the backing plate (aluminum) thickness. In the same table are also shown approximate values for the backscattering saturation thickness. An empirical model of $\eta(x)$ was obtained by least square fitting of data from Table 6 with the following function

$$\eta(x) = \frac{\alpha x}{1 + \beta x}$$  \hspace{1cm} (9)

The values of fitting parameters $\alpha$ and $\beta$ corresponding to $^{14}$C, $^{147}$Pm, $^{60}$Co, $^{137}$Cs, $^{36}$Cl, and $^{90}$Sr-90Y are shown in Table 7.

The backscattering coefficient at saturation is of practical interest and therefore it was calculated for different materials of the backscattering coefficient, $\eta(x)$, for different values, $x$, of the backing plate (aluminum) thickness. In the same table are also shown approximate values for the backscattering saturation thickness. An empirical model of $\eta(x)$ was obtained by least square fitting of data from Table 6 with the following function

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The values of fitting parameters $\alpha$ and $\beta$ corresponding to $^{14}$C, $^{147}$Pm, $^{60}$Co, $^{137}$Cs, $^{36}$Cl, and $^{90}$Sr-90Y are shown in Table 7.

The backscattering coefficient at saturation was measured in planar geometry (2\textpi solid angle) for different materials of the backing plate (Lucite, aluminum, copper, silver and lead) using a 4\textpi beta counter and a $^{32}$P source ($E_{\text{max}} = 1.71$ MeV). Discrepancies smaller than 10% were found between the results reported by Seliger in Fig. 2 of the paper (Seliger, 1952) and the results obtained in this work using Eq. (10). These discrepancies are due to the errors of calculation given by Eq. (10) and the uncertainty in the measurement of the saturation backscattering coefficient.

Though the reliability of the Pencyl code for electron transport was previously mentioned, the above comparisons give more confidence in the results obtained in this paper.

5. Applications

5.1. Efficiency of large-area beta sources

The surface emission rate, $E_{\text{s}}$, of the large-area source shown in Fig. 1 (covering foil is neglected) is defined by ISO 8769 and can be calculated by means of Eq. (2). Thus, we get

$$\eta_{\text{sat}}(x) = \eta_{\text{sat}}(x) + (\eta_{\text{sat}}(x) - \eta_{\text{sat}}(x)) \exp(-\chi/\theta)$$  \hspace{1cm} (11)

where $\eta_{\text{sat}}$ and $\eta_{\text{sat}}$ are, respectively, the saturation backscattering coefficient of the aluminum plate and the Mylar foil and $\theta$ is the fitting parameter which is also shown in Table 9.

4. Comparison with data found in the literature

To check the validity of the models presented in this paper, we compare the Monte Carlo data obtained from Pencyl code with data from literature. Thus, the results reported by Berger in Table 2 of the paper (Berger, 1998) are compared with the Monte Carlo results given in Table 1 (aluminum) of this paper. The maximum discrepancy between results is 3.3% obtained in the case of the nuclide $^{14}$C for $x = 2.16$ mg/cm$^2$. Although the TRANSIT code used by Berger and Pencyl code have different interaction models and tracking algorithms, they give results in very good agreement.

In the paper (Seliger, 1952), the backscattering coefficient at saturation was measured in planar geometry (2\textpi solid angle) for different materials of the backing plate (Lucite, aluminum, copper, silver and lead) using a 4\textpi beta counter and a $^{32}$P source ($E_{\text{max}} = 1.71$ MeV). Discrepancies smaller than 10% were found between the results reported by Seliger in Fig. 2 of the paper (Seliger, 1952) and the results obtained in this work using Eq. (10). These discrepancies are due to the errors of calculation given by Eq. (10) and the uncertainty in the measurement of the saturation backscattering coefficient.

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### Table 4

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$^{14}$C</th>
<th>$^{147}$Pm</th>
<th>$^{60}$Co</th>
<th>$^{36}$Cl</th>
<th>$^{90}$Sr-90Y</th>
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</thead>
<tbody>
<tr>
<td>Backing plate: aluminum</td>
<td>Source substrate: aluminum</td>
<td>$a_0$</td>
<td>$a_1$ (cm/mg$^{-0.5}$)</td>
<td>$a_2$ (cm/mg)</td>
<td>$a_3$ (cm$^3$/mg$^{1.5}$)</td>
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<tr>
<td>$a_0$</td>
<td>0.721095400</td>
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<td>0.709496911</td>
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<tr>
<td>$a_1$ (cm/mg$^{-0.5}$)</td>
<td>-0.554508690</td>
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<td>-0.340266940</td>
<td>-0.115271820</td>
<td>-0.098614630</td>
</tr>
<tr>
<td>$a_2$ (cm/mg)</td>
<td>0.121388777</td>
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<td>$a_3$ (cm$^3$/mg$^{1.5}$)</td>
<td>0.019580022</td>
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<td>$a_4$ (cm$^4$/mg$^2$)</td>
<td>-0.014001100</td>
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<td>0.000024287</td>
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<td>$a_5$ (cm$^5$/mg$^{2.5}$)</td>
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<td>-0.000318940</td>
<td>0.0000376</td>
<td>0.000009050</td>
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### Table 5

<table>
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<tr>
<th>Materials</th>
<th>$a_0$</th>
<th>$a_1$ (cm/mg$^{-0.5}$)</th>
<th>$a_2$ (cm/mg)</th>
<th>$a_3$ (cm$^3$/mg$^{1.5}$)</th>
<th>$a_4$ (cm$^4$/mg$^2$)</th>
<th>$a_5$ (cm$^5$/mg$^{2.5}$)</th>
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<td>Aluminum</td>
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<td>-0.003506495</td>
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<tr>
<td>Mylar</td>
<td>0.665848407</td>
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<td>-0.004140524</td>
<td>0.000539681</td>
<td>-0.000027459</td>
</tr>
</tbody>
</table>
where $A$ is the source activity, $f(x) = A \Delta p(x)/A$ is the activity depth distribution and $x_{\text{max}}$ represents the thickness of the active layer of the source which may be smaller than the thickness of the source material (see Fig. 1). Eq. (12) shows that there is a simple relationship between $E_s$ and the activity, $A$, of the source and $A$ can be calculated for a given value of $E_s$ provided that $x_{\text{max}}$ and $f(x)$ are known.

The efficiency, $E_s$, of large-area beta sources is defined as the fraction of the emitted particles that emerge from the top surface of the source in a 2$\pi$ solid angle. It follows that

$$E_s = \frac{E_s}{\pi A} = \int_0^{x_{\text{max}}} \varepsilon p(x, \Delta) f(x) \, dx$$

(13)

In case that the same material is used for the backing plate and source substrate, $\varepsilon p(x, \Delta)$ from Eqs. (12) and (13) must be replaced with $\varepsilon p(x)$ (see Eq. (11)).

Taking into account that the source is covered by a foil of thickness $s$ as it is shown in Fig. 1, the emission rate in 2$\pi$ of the large-area source (the rate of beta particles emitted by the large-area source that emerge from the top surface of the covering foil in

Table 7

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$^{14}$C</th>
<th>$^{147}$Pm</th>
<th>$^{60}$Co</th>
<th>$^{137}$Cs</th>
<th>$^{36}$Cl</th>
<th>$^{90}$Sr-$^{90}$Y</th>
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<td>$\alpha$ (cm/mg$^{0.5}$)</td>
<td>1.50</td>
<td>1.29</td>
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<td>0.50</td>
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<tr>
<td>$\beta$ (cm/mg$^{0.5}$)</td>
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<td>2.53</td>
<td>1.64</td>
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Table 8

<table>
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<th>Backing plate</th>
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<th>Saturation backscattering coefficient</th>
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<td>$^{147}$Pm</td>
<td>$^{60}$Co</td>
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<td>Beryllium</td>
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Table 6

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<tr>
<th>$^{14}$C</th>
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<th>$^{60}$Co</th>
<th>$^{137}$Cs</th>
<th>$^{36}$Cl</th>
<th>$^{90}$Sr-$^{90}$Y</th>
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<td>$x$ (mg/cm$^2$)</td>
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<td>$x$ (mg/cm$^2$)</td>
<td>$\eta(x)$ (cm/mg$^{0.5}$)</td>
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<td>8.100</td>
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<td>0.436</td>
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Backscattering saturation thickness (mg/cm$^2$)

<table>
<thead>
<tr>
<th>$^{14}$C</th>
<th>$^{147}$Pm</th>
<th>$^{60}$Co</th>
<th>$^{137}$Cs</th>
<th>$^{36}$Cl</th>
<th>$^{90}$Sr-$^{90}$Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$ (mg/cm$^2$)</td>
<td>$\eta(x)$ (cm/mg$^{0.5}$)</td>
<td>$x$ (mg/cm$^2$)</td>
<td>$\eta(x)$ (cm/mg$^{0.5}$)</td>
<td>$x$ (mg/cm$^2$)</td>
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</tr>
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<td>6.75</td>
<td>6.75</td>
<td>13.50</td>
<td>32.40</td>
<td>48.60</td>
<td>216.00</td>
</tr>
</tbody>
</table>
a 2π solid angle, $E(s)$, can be calculated by means of Eq. (5). Thus, we have

$$E(s) = \int_0^{\varepsilon_{\text{max}}} \varepsilon_p(x, \Delta, s) A_p(x)dx$$

$$= \int_0^{\varepsilon_{\text{max}}} \varepsilon_p(x, \Delta, s) f(x)dx$$  \hspace{1cm} (14)

The efficiency of the large-area source is given by

$$e(s) = \frac{E(s)}{f_p A} = \int_0^{\varepsilon_{\text{max}}} \varepsilon_p(x, \Delta, s) f(x)dx$$  \hspace{1cm} (15)

The transmission coefficient of beta particles emitted by the large-area source through the covering foil of thickness $s$ is defined as the fraction of beta particles transmitted through this foil. As a result, we have

$$t(s) = \frac{E(s)}{E_0} = \int_0^{\varepsilon_{\text{max}}} \varepsilon_p(x, \Delta, s) f(x)dx$$

(16)

One can prove that $\varepsilon_p(x, \Delta, s)$ from Eqs. (14), (15) and (16) must be replaced with $\varepsilon_p(x + s)$ if the covering foil, source substrate and backing plate are made from the same material. If only the covering foil and the source substrate are made from the same material, one can also be proved that $\varepsilon_p(x, \Delta, s)$ from Eqs. (14), (15) and (16) must be replaced with $\varepsilon_p(x + s, \Delta + s)$.

5.2. Efficiency of large-area beta sources fabricated by the ink-jet printing technique

As an example of application, we calculate the efficiency of $^{36}$Cl large-area reference sources fabricated by the ink-jet printing technique (Yamada et al., 2012). A thin layer of about 5 μm containing the radioactive material is applied by ink-jet printing technique on a substrate (polyester foil) of 0.1 mm thickness (14 mg/cm²) which is mounted in an aluminum frame (4 mm thickness). To avoid radioactivity contamination, the source is covered with a 0.9 mg/cm² aluminized Mylar film. Because the effective atomic numbers of the polyester foil and the radioactive material are very close to the effective atomic number of the Mylar foil, we consider these materials as Mylar.

The source efficiency is calculated using Eq. (15), taking $x_{\text{max}} = 0.7$ mg/cm², $s = 0.9$ mg/cm², $\Delta = 14.7$ mg/cm², assuming a homogeneous activity distribution ($f(x) = 1/x_{\text{max}}$) and knowing that $\varepsilon_p(x, \Delta, s) = \varepsilon_p(x + s, \Delta + s) = a_0 + a_1(x + s)^5 + a_2(x + s)^3 + a_3(x + s)^2 + a_4(x + s)$.

The fitting parameters $a_0, a_1, a_2, a_3$ and $a_4$ were previously calculated at the end of the subchapter 2.3. Consequently, the calculated value of the source efficiency is 0.563. The experimental value of the source efficiency reported by Yamada et al. (2012) was 0.551. As one can see, there is a very good agreement between the experimental value and the calculated value of the source efficiency. This good agreement proves the usefulness of the plane source concept and the models developed for calculating the efficiency of large-area beta sources.

5.3. Detection efficiency of large-area detectors

For the sake of simplicity, we consider the planar geometry shown in Fig. 4 where the backing plate, the source stratum and the detector window are made from the same material. This geometry refers to the detection of beta particles using large-area detectors. The detection efficiency, $\epsilon_{\text{det}}$, is defined as the probability of detecting and recording beta particles by the counting system for a given counting geometry and detector (ICRU, 1994).

As a result, we have

$$\epsilon_{\text{det}} = \frac{R}{f_p A}$$  \hspace{1cm} (17)

where $R$ is the count rate recorded by the counting system corrected for background, dead time losses and decay, $A$ is the activity of the source. It is evident that $R = p_0 E(s)$  \hspace{1cm} (18)

where $p_0$ is the probability of detecting and recording of beta particles reaching the sensible volume of the detector and $E(s)$ is the rate of these particles. Knowing that $E(s) = t_{\text{exp}} E_0 = t_{\text{exp}} f_p A$ ($t_{\text{exp}}$ is the transmission coefficient of beta particle through the detector window), it follows that

$$\epsilon_{\text{det}} = p_0 \varepsilon_p s_{\text{det}}$$  \hspace{1cm} (19)

where $s_{\text{det}} = p_0 f_p$ is the instrument efficiency defined by ISO 7503-1. Using Eqs. (17), (18) and (14), we get

$$\epsilon_{\text{det}} = p_0 \int_0^{\varepsilon_{\text{max}}} \varepsilon_p(x + s) f(x)dx = p_0 \left[ \varepsilon_p(s) - I(x_{\text{max}}, s) \right]$$  \hspace{1cm} (20)

Fig. 4. Planar geometry for the detection of beta particles using large-area detectors.
where $I(x_{\text{max}}) = \int_0^{x_{\text{max}}} (\rho_x(s) - \rho_x(x + s))f(x)dx$. As one can see from Eq. (20), the maximum value of the detection efficiency is $\epsilon_{\text{det}}^{\max} = \rho_x(0) = 0.5\rho_x (1 + \eta_s)$ obtained for $x_{\text{max}} = 0$ and $s = 0$ ($\eta_s$ is the saturation backscattering coefficient of the backing plate material). This means that the maximum of the detection efficiency is obtained by using windowless detectors and sources of negligible thickness deposited on materials having high atomic numbers (e.g., tungsten). It should be noted that $I(x_{\text{max}})$ is usually much smaller than $\rho_x(s)$ (especially for high energy beta emitters such as $^{137}$Cs, $^{36}$Cl and $^{86}$Sr-$^{88}$Y) for $s > x_{\text{max}}$, which means that the detection efficiency depends weakly on $x_{\text{max}}$ and $f(x)$ under this condition. Consequently, surface contamination measurements are less susceptible to fluctuations of the contamination thickness and the depth activity distribution by using thick detector windows ($s > x_{\text{max}}$) but the detection efficiency is significantly decreased in this case.

6. Conclusions

The modeling of the transmission of beta-rays through thin foils in planar geometry was performed using the plane source concept. The Pencyl code from the simulation package PENELOPE was used for computing the plane source efficiency and backscattering coefficients. Simple analytical expressions were obtained for the efficiency of plane sources and backscattering coefficients using least squares fitting of Monte Carlo data.

The utility of the modeling of the transmission of beta-rays through foils in planar geometry was proved in two applications regarding the efficiency of large-area beta sources and the detection efficiency of large-area detectors used for surface contamination measurements. Thus, the surface emission rate and the efficiency of large-area beta sources were defined by means of the plane source efficiency. It is shown that there is a simple relationship between surface emission rate of the source $E_s$ and its activity $A$. As an example of application, the efficiency of large-area reference sources fabricated by the ink-jet printing technique was calculated. The detection efficiency of large-area detectors used for surface beta contamination measurements was also evaluated using the plane source concept. It is shown that the detection efficiency depends on the source efficiency and the transmission coefficient of beta radiation through the detector window. It is also shown that surface contamination measurements are less susceptible to fluctuations of the contamination thickness and the activity depth distribution by using thick detector windows.

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References


