Sampling based on Bayesian statistics and scaling factors

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MetroDECOM Task 2

• D1: Currently employed sampling strategies reviewed and reported
  - Complete

• D2: Report on Bayesian analysis of sampling requirements identified in 1.2.1
  - Reported here

• D3: Report on strategy to minimise sampling without risking false negatives
  - Not started

• D4: Report summarising statistical analysis and sampling strategies written
  - Not started
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LII. An Essay towards solving a Problem in the Doctrine of Chances. By the late Rev. Mr. Bayes, Price, in Rant, A. M. F. R. S.

Dear Sir,

Read Dec. 23, 1763.

I am much obliged by your letter, in which you inform me that the paper which I have sent you, in my opinion, has great merit, and well deserves to be preserved. Experimental philosophy, you will find, is nearly interested in the subject of it; and on this account there
Bayes theorem

- **What he said:**
  - ‘The probability of any event is the ratio between the value at which an expectation depending on the happening of the event ought to be computed, and the value of the thing expected upon its happening’
  
  Quite.

- **In reality:**

  \[ P(A|B) = \frac{P(B|A)P(A)}{P(B)} \]

  - \( A \) and \( B \) are events
  - \( P(A) \) and \( P(B) \) are the probability of observing \( A \) and \( B \)
  - \( P(A|B) \) is the probability of observing \( A \), given \( B \) is true
  - \( P(B|A) \) is the probability of observing \( B \), given \( A \) is true

  Quite, as any fule kno.
Consider a woman who has a brother with haemophilia; her father does not suffer from haemophilia.

She either carries \(A\), or does not carry \(B\), the haemophilia gene, so, based on this information:

\[ P(A) = P(B) = 0.5 \]

She has two non-identical non haemophiliac sons:

\[ P(s_1 = 0, s_2 = 0 | A) = 0.25 \]

- The probability of having a healthy son is 0.5 if she is a carrier.

\[ P(s_1 = 0, s_2 = 0 | B) = 1 \]

- The probability of having a healthy son is 1 if she is a not a carrier, ignoring random mutations of this gene.
So what is the probability of the woman carrying the haemophilia gene?

\[ P(A|y) = \frac{P(y|A)P(A)}{P(y|A)P(A) + P(y|B)P(B)} \]

- In numbers:

\[ P(A|y) = \frac{0.25 \times 0.5}{0.25 \times 0.5 + 1 \times 0.5} = \frac{0.125}{0.625} = 0.2 \]

- If she has a third unaffected son:

\[ P(A|y) = \frac{0.5 \times 0.2}{0.5 \times 0.2 + 1 \times (1 - 0.2)} = \frac{0.1}{0.9} = 0.111 \ldots \]

- If her third son is affected, then:

\[ P(A) = 1; P(B) = 0 \]

Quite easy, really.
# Bayesian analysis

<table>
<thead>
<tr>
<th></th>
<th>Classical</th>
<th>Bayesian</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distribution</strong></td>
<td>$f(y</td>
<td>\eta)$</td>
</tr>
<tr>
<td><strong>Meaning</strong></td>
<td>Distribution of the measurement result, $y$, given the true value, $\eta$, of the measurand $Y$</td>
<td>Distribution of the true value, $\eta$, of the measurand $Y$ given measurement result, $y$</td>
</tr>
</tbody>
</table>

$$f(\eta|y) = C f_0(\eta|y)f(\eta)$$
• What are our prior distributions?
  - What do we know about the history of the potentially contaminated area?
  - Information from α-emissions
    Very localised, and within <100 μm of the surface
    Background is near to zero
  - Information from β-emissions
    Localised, but information from up to 1 cm deep, depending on the emitter and the material
    Background must be subtracted
  - Information from γ-emissions
    Diffuse, but information from many centimetres deep, depending on the emitter and the material
    Background must be subtracted

• Mapping
  - This information allows a crude contamination map to be drawn
Assessing contamination (II)

- These priors direct initial sampling
  - Feedback the results of sample measurement
    - Modifies the probability ‘map’ of the surface
    - Redirects sampling effort
  - Approach is identical (in concept) to the example
    - The mathematics is rather more complex
    - Better done by software.
  - What it will not do
    - Identify hot-spots of pure $\alpha$- or pure $\beta$-emitters buried beneath the surface
    - Unlikely to occur without associated $\gamma$-emitters
    - May observe bremsstrahlung to reveal such events
Conclusion

- Bayesian inference is a powerful tool
  - Can better direct effort, if employed correctly
  - But it is not necessarily simple
- Involve persons with a good knowledge of the technique
- Report outlines the general principles that can be used
- Actual use is entirely dependent on the site being monitored
SCALE FACTOR INVESTIGATION
Scaling Factors (SFs) – Task 1.4

- **Scaling Factors (SFs):** instrument for determining **Hard To Measure Radionuclides (HTMR).**
- Basic idea: radionuclides from a same source and behaviour (speciation, charge, solubility, volatility, etc) maintain a constant relationship in their final activity concentrations.
- Correlations between HTMR and an **Easy To Measure (ETM) radionuclide,** called also **Key Nuclide (KN).**
- Statistical validation needed for valid SFs.
- **MetroDECOM** project will lead to better quality SFs for use in future decommissioning activities within the EU.
- Derive new and better scaling factors.
# Scaling Factors deliverables

<table>
<thead>
<tr>
<th>Del. No.</th>
<th>Deliverable description</th>
<th>Lead JRP-Partner</th>
<th>Other JRP-Participants</th>
<th>Delivery date</th>
<th>Status and activity (one paragraph includes all JRP-Participants)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4.1</td>
<td>Relevant scaling factor databases identified</td>
<td>ENEA</td>
<td>CMI, NPL</td>
<td>Nov 2014</td>
<td><strong>This Deliverable is complete.</strong></td>
</tr>
<tr>
<td>1.4.2</td>
<td>Report on evaluation of existing data for the radionuclides</td>
<td>ENEA</td>
<td>CMI, NPL</td>
<td>Aug 2015</td>
<td><strong>This Deliverable is complete.</strong></td>
</tr>
<tr>
<td>1.4.3</td>
<td>Report on the radionuclide list prioritised for further study of up to 10 radionuclides that have scaling factors that are either poorly known or not determined</td>
<td>ENEA</td>
<td>CMI, NPL</td>
<td>Nov 2015</td>
<td><strong>This deliverable must be completed. Work in progress. A new draft was prepared on this task by taking into account the results achieved in D1.4.1 and D.1.4.2</strong></td>
</tr>
<tr>
<td>1.4.4</td>
<td>Report on initial or improved scaling factors for the selected radionuclides</td>
<td>ENEA</td>
<td>CMI, NPL</td>
<td>Aug 2016</td>
<td><strong>Work has not yet started on this deliverable.</strong></td>
</tr>
<tr>
<td>1.4.5</td>
<td>Report on initial or improved optimal scaling factors for the selected radionuclides</td>
<td>ENEA</td>
<td>CMI, NPL</td>
<td>May 2017</td>
<td><strong>Work has not yet started on this deliverable.</strong></td>
</tr>
<tr>
<td>1.4.6</td>
<td>Good Practice Guide for Radionuclide Scaling Factors, based on ISO 21238:2007 produced</td>
<td>NPL</td>
<td>CMI, ENEA</td>
<td>Feb 2017</td>
<td><strong>Work has not yet started on this deliverable.</strong></td>
</tr>
<tr>
<td>1.4.7</td>
<td>Report on scaling factors specific to BR3 decommissioning produced.</td>
<td>NPL</td>
<td>CMI, ENEA, SCK-CEN</td>
<td>Feb 2017</td>
<td><strong>Work has not yet started on this deliverable.</strong></td>
</tr>
<tr>
<td>1.4.8</td>
<td>Paper on scaling factors submitted for publication in a peer-reviewed journal</td>
<td>NPL</td>
<td>CMI, ENEA, SCK-CEN</td>
<td>Jun 2017</td>
<td><strong>Work has not yet started on this deliverable.</strong></td>
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<td>1.4.9</td>
<td>Final summary report written</td>
<td>NPL</td>
<td>CMI, ENEA</td>
<td>Jun 2017</td>
<td><strong>Work has not yet started on this deliverable.</strong></td>
</tr>
</tbody>
</table>
Scale Factors data bases

- NUREG/CR-4101 “Assay of Long-Lived Radionuclides in Low-Level Wastes From Power Reactors”.
- EPRI (USA):
  - Updated Scaling Factors in Low-Level Radwaste (1987);
- BR3 RNvector contamination 19980701.pdf
SF findings

• γ emitters, Co-60, Ba-133, Cs-137, Eu-152 and Eu154 easily measured through γ-spectrometry (NDA technique).
• α and β-emitters need of dedicated samples preparation (DA technique).
• SFs for radioactive materials of different origin and composition available in literature but limited to:
  • large variability of SFs because of different origin of the materials;
  • differences existing between different plant and/or use of the same plant.
• Literature SFs useful in screening operations from which only general indications can be derived.
• Targeted SFs assessments needed through experimental investigations with greater accuracy.
• MetroDECOM: SFs for 3 reference materials: concrete, steel and graphite.
The Key Nuclide (KN)

- Fundamental is the choice of the KN.
- KN is selected according to very precise characteristics:
  - easily measured non-destructively (this implies, in many cases, that it is a strong γ-emitter);
  - chemical and physical properties similar to the HTMR to be identified and/or studied.
- Normally only two radionuclides (Co-60 and the Cs-137) may be considered appropriate as KNs.
Evaluation of existing data

• General aspects

• In order to have a real correlation between the two radionuclides the following conditions must be accomplished:
  • KN activity concentration measurable and far above the MDA;
  • samples representativeness of all materials;
  • samples representativeness of all plant areas.
  • repeated measurements fore statistical significance;
  • different waste streams or plant types comparability;
  • functional relationship between the measured activity of the two radionuclides.

• Scaling factor:
  \[
  SF \left( \frac{A}{B} \right) = \frac{a_s(A)}{a_s(B)}
  \]

• Linear regression:
  \[
  A_{RN} = a + b \cdot A_{KN}
  \]

• Log-normal distribution (log linear regression)
  \[
  \ln(A_{RN}) = d + f \cdot \ln(A_{KN})
  \]
  \[
  A_{RN} = h \cdot (A_{KN})^f
  \]
Evaluation of existing data

- **Computation methods**

- For the HTMR activation products (Fe-55, Ni-63, C-14) SFs theoretical estimate possible.

- These radionuclides come from the non active radionuclides (Fe-54, Ni-62 e Co-59) activated by thermal neutrons.

- Activation products:

\[
\frac{dN}{dt} = \sigma \cdot M - \lambda \cdot N - \varepsilon \cdot N
\]

\[
A_2 = \frac{\sigma_2 \lambda_2 M_2}{\sigma_1 \lambda_1 M_1} A_1
\]

- Burn up:

\[
\frac{dN_i}{dt} = \sum_j \gamma_{ji} \sigma_i \beta_j N_j + \sigma_{c,i-1} N_{i-1} + \sum_k \lambda_k N_k - \sigma_{f,i} N_i - \sigma_{c,i} N_i - \lambda_i N_i
\]
Evaluation of existing data

• **Summary**

• The SFs are dependent from:
  - type of nuclear plant and reactor (BWR, PWR, etc.);
  - composition of the materials (impurities);
  - neutron flux (intensity and neutron energy);
  - cross section of the reactions (activation, fission, etc.);
  - source-target geometry;
  - live time of the nuclear plant;
  - dead time since shutdown and starting of the analysis data.

• Difficult to find SFs for concrete, graphite and steel.

• By using known data: SFs for Stainless Steel material (USA):

<table>
<thead>
<tr>
<th></th>
<th>36Cl/60Co</th>
<th>93Mo/60Co</th>
<th>113mCd/60Co</th>
</tr>
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<tbody>
<tr>
<td>from</td>
<td>&lt;8.1 E-7</td>
<td>1.5 E-7</td>
<td>5.0 E-5</td>
</tr>
<tr>
<td>to</td>
<td>&lt;1.5 E-4</td>
<td>1.1 E-5</td>
<td>&lt;3.3 E-4</td>
</tr>
</tbody>
</table>

N. 20/20
Evaluation of existing data

• **Considerations on steel**
  • Many different types of steel – even well specified steels have impurities.
  • Often the impurities have the high neutron capture cross sections.
  • Time or neutron spectrum to which the steel was exposed are poorly known.
  • **Generic scaling factors for steel are difficult (probably impossible).**
Prosecution of work

- Prioritized radionuclides
- Nuclides selected as HDM and with SFs poorly known and not determined:

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>Radiation</th>
<th>Principal Source</th>
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<tbody>
<tr>
<td>$^{85}\text{Sr}$</td>
<td>EC</td>
<td>Activation</td>
</tr>
<tr>
<td>$^{90}\text{Sr}$</td>
<td>$\beta$-</td>
<td>Fission Product</td>
</tr>
<tr>
<td>$^{93}\text{Zr}$</td>
<td>$\beta$-</td>
<td>Activation $^{92}\text{Zr}$</td>
</tr>
<tr>
<td>$^{151}\text{Sm}$</td>
<td>$\beta$, $\gamma$</td>
<td>Fission Product</td>
</tr>
<tr>
<td>$^{226}\text{Ra}$</td>
<td>$\alpha$, $\gamma$</td>
<td>Decay $^{238}\text{U}$</td>
</tr>
<tr>
<td>$^{234}\text{U}$</td>
<td>$\alpha$</td>
<td>Naturally occurring</td>
</tr>
<tr>
<td>$^{235}\text{U}$</td>
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</tr>
<tr>
<td>$^{236}\text{U}$</td>
<td>$\alpha$</td>
<td>Activation $^{235}\text{U}$</td>
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<tr>
<td>$^{238}\text{U}$</td>
<td>$\alpha$</td>
<td>Naturally occurring</td>
</tr>
<tr>
<td>$^{237}\text{Np}$</td>
<td>$\alpha$</td>
<td>Neutron capture</td>
</tr>
<tr>
<td>$^{238}\text{Pu}$</td>
<td>$\alpha$</td>
<td>Decay $^{238}\text{Np}$</td>
</tr>
<tr>
<td>$^{239}\text{Pu}$</td>
<td>$\alpha$</td>
<td>Activation $^{238}\text{U}$</td>
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<tr>
<td>$^{240}\text{Pu}$</td>
<td>$\alpha$</td>
<td>Activation $^{239}\text{Pu}$</td>
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<tr>
<td>$^{241}\text{Pu}$</td>
<td>$\beta$-</td>
<td>Activation $^{240}\text{Pu}$</td>
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</table>
Thank you